of the American Philosophical Society because it had chosen Franklin for its President, and that Richard Penn had been Lieutenant Governor (as Deputy for that uncle and his brother) from 1771 to 1773, it must have been difficult for Franklin not to feel that such a letter from such a man was indeed a tribute to his position, achieved solely by his own efforts.

From this mass of correspondence, I have selected some letters showing the state of public opinion in New England in 1774, and from London in 1775, including a characteristic letter from Priestley and from Charles Lee and Wayne in the field. Much more might be printed to show how well Franklin kept in touch with all that was of interest during his long and busy career. It is well that this venerable Society, so largely the result of his labors, should be made the custodian of the papers that follow almost his daily thoughts, and it is to be hoped that the preparation and publication of a Calendar showing their contents may be completed at no distant day, certainly by the two hundredth anniversary of the birth of our founder, and thus perpetuate his memory.

Franklin's legacy to the Philosophical Society was ninety-one volumes of the *History of the Royal Academy of Sciences* at Paris, thus helping that collection of publications of scientific societies that make so valuable a portion of its Library.

THE ORBIT OF THE DOUBLE STAR Σ 518.

BY ERIC DOOLITTLE.

(*Read April 3, 1903.*)

INTRODUCTION.

It is well known to astronomers that many of the stars of the sky which to the naked eye appear to be but single stars are when viewed with the telescope seen to be made up of two or more stars very close together. About twenty thousand such double stars have been measured and catalogued, and the number is continually being added to through the discoveries of the great modern telescopes. There are scarcely fifty of these, however, of which a determination of the orbit is possible.

It was in the years 1802 and 1803 that the classic memoirs of Herschel appeared, in which it was shown for the first time that

the two stars of a binary system revolve in elliptic orbits about their common center of gravity. The first method for determining the orbit of the companion star about its primary was given by Savary in 1827, who applied his method to the binary ξ Ursæ Majoris. This was thus the first double star of which an orbit was computed.

In the method of Savary, the elements of the orbit were derived from the least possible number of measures which would theoretically determine them. It was thus but very poorly adapted to secure good results, since all double star measures are liable to errors which are very large in proportion to the quantities to be determined from them. The method was improved by Encke, and other methods were subsequently devised by Sir John Herschel, Villarceau, Thiele and others; but in all of these the development was from the point of view of the pure mathematician, rather than from that of the practical astronomer.

The astronomer who essays to compute the orbit of a double star will find that he has at hand a great mass of measures, which, having been made by observers of varying experience and with instruments of all degrees of perfection, are more or less discordant. Each one of these measures consists of a determination at a given time of the distance and direction of the companion star from its primary.

If now these measures be plotted, by taking a point on the paper to represent the principal star and laying off from this point each measured distance and direction to the companion star, a series of other points will be obtained which will represent to the eye the path which the companion has pursued about its primary. Were the measures free from error, the points which indicate the position of the companion would lie accuratly upon the perimeter of an ellipse; but practically they are very far from doing so, especially if the double star is very close and difficult of measurement.

The ellipse which the companion appears to describe does not represent the true orbit of the body in space, since the true orbit is viewed more or less obliquely. It is evidently the projection of the true orbit on a plane tangent to the celestial sphere at the point at which the double star is situated. While the true orbit in space is an ellipse of which the principal star occupies the focus, the apparent or projected orbit, though also necessarily an ellipse, will not have its focus at the principal star. Nevertheless, Kepler's

second Law, which states that the areas swept over by the radius-vector are proportional to the corresponding times, will evidently be true, provided that in the apparent orbit these radii-vectores are drawn from the principal star instead of from the focus.

Having plotted the series of measures as above described, the first step in the determination of a double star orbit is to draw the apparent ellipse in such a manner that it shall represent them reasonably well; the various sectorial areas are then measured with a planimeter, or otherwise, and the trial ellipse changed in shape and position until finally, after several trials, the measured positions and the law of areas are both approximately satisfied.

To fix the shape of the true orbit and its position in space, and to predict the future motion, there must next be determined the following seven elements:

(1) *The Period, P.* This can be measured directly from the apparent ellipse, since, by Kepler's Law, any sectorial area is to that of the whole ellipse as the time occupied in the description of the area is to the Period.

(2) *The Time of Periastron Passage, T.* This is the date at which the companion passes the nearer vertex of the true ellipse. It can evidently be found from the apparent ellipse by an application of Kepler's Law.

(3) *The Eccentricity, e.* This, since it is a ratio, can be obtained from the apparent ellipse.

(4) *The Inclination, i,* of the true orbit to the tangent plane.

(5) *The Longitude, Ω,* of the intersection of two planes.

(6) *The Longitude, λ,* of periastron.

The last three elements are obtained by solving a spherical triangle. The longitudes are measured from the hour circle passing through the star, from the north point in the direction of motion.

(7.) *The Semi-Major Axis, a.*

The elements of the true orbit as thus obtained enable us to predict the direction and distance of the companion for any time. The next step of the computation is to obtain the computed distance and direction at the date of each observation. A comparison of the computed with the observed positions furnishes a basis for improving the elements by the principles of Least Squares. The same process is repeated with the improved elements, until a satisfactory agreement between the computed and observed positions is obtained.

THE COMPUTATION.

There are available for this determination measures on 141 nights, as shown in the following table. In the first column will be found the date of observation; in the second, the measured distance; in the third, the measured angle; in the fourth, the number of nights on which the measures were made, and in the fifth, the name of the observer.

	Date.	*ρ.*	*θ.*	*n.*	*Observer.*
1	1783.13	″ ″ 4 to 8	° 326.7	1	Herschel.
2	1825.12		287 ±	1	Struve.
3	1835 to '36				"
4	1850.94	″ 3.96	156.60	2	Otto Struve.
5	1851.06	3 ±	159.96	1	Dawes.
6	1851.49	3.87	155.10	2	Otto Struve.
7	1853.64	3.93	158.30	3	"
8	1854.79	4.13	155.30	1	"
9	1856.80	4.51	152.90	1	"
10	1857.82	4.40	153.00	1	"
11	1864.84	4.45	147.60	2	Winnecke.
12	1865.89	4.26	143.95	2	Otto Struve.
13	1869.10	4.46	140.40	1	"
14	1871.99	2 ±	125 ±	1	Knott.
15	1872.56	4.62	140.65	1–2	Otto Struve.
16	1873.99	4.27	133.90	1	"
17	1874.10	4.39	135.70	1	"
18	1875.14	3.80	138.10	1	"
19	1876.11	4.01	130.50	1	"
20	1877.12	2 ±	120.	1	Flammarion.
21	1877.84	3.36	127.50	3	Cincinnati.
22	1877.84	3.92	128.24	6	Burnham.
23	1877.95	3.94	126.45	4	Dembowski.
24	1878.14	4.36	125.50	1	Otto Struve.
25	1879.05	3.49	125.38	4	Burnham.
26	1879.18	3.52	125.00	2	Hall.
27	1879.75	3.29	120.00	1	Cincinnati.
28	1880.09	3.28	121.30	5	Burnham.
29	1880.95	3.16	122.06	5	"
30	1881.84	3.53	119.00	6	"
31	1882.12	3.25	118.15	2	Hall.
32	1883.00	3.07	119.20	2	Burnham.
33	1883.81	3.10	115.80	2	Hall.
34	1884.16	3.74	118.20	1	Herman Struve.
35	1886.00	3.22	112.15	2	Leavenworth.

	Date.	*ρ.*	*θ.*	*n.*	*Observer.*
		″	°		
36	1886.09	3.00	112.23	6	Hall.
37	1886.92	3.01	111.03	3	Tarrant.
38	1887.14	2.56	109.18	1–4	Schiaparelli.
39	1888.08	2.26	109.48	2	"
40	1888.12	3.04	107.68	5	Hall.
41	1888.84	2.94	106.83	3	Burnham.
42	1888.87	2.81	105.05	3	Tarrant.
43	1889.03	2.87	107.59	1–2	Schiaparelli.
44	1889.12	2.79	103.55	4	Hall.
45	1890.73	2.68	99.95	4	Burnham.
46	1890.98	1.72	99.00	3	Hough.
47	1891.01	2.62	101.49	2	Schiaparelli.
48	1891.06	2.65	98.56	5	Hall.
49	1891.78	2.48	97.38	4	Burnham.
50	1893.21	2.18	93.8	1	Comstock.
51	1895.89		83.65	0–1	Doberck.
52	1895.91	2.32	87.4	1	Collins.
53	1897.97	2.62	77.22	3	Aitken.
54	1899.11	2.39	73.6	2	"
55	1899.80	2.30	68.35	3	Doolittle.
56	1900.92	2.40	63.41	2	"
57	1903.14	2.34	55.22	4	"

NOTES—(1) Herschel placed the pair in his "Class II," which indicates that he estimated the distance as between 4″ and 8″. Otto Struve considers that at this time the distance must have been less than 4″, which seems the more probable. No use has been made of this measure in the final adjustment. (2) Excessively difficult. The angle was estimated roughly as being in the direction of the principal star, of which the position angle is 107°. The entire unreliability of this measure was first pointed out by Burnham in 1894. (3) No trace of duplicity. (14) This is merely a rough estimate. Knott used a 7⅓ inch refractor. (51) "Nearly invisible." (53) and (54) Made with the 12-inch. I have given half the theoretical weight to numbers (5), (38) and (43). (57) Was not used in the computation; these observations were made after the work was completed.

These observations were corrected for precession, and then plotted as above described, and the elements of the true orbit were derived from them. These elements were the following:

$$\left.\begin{aligned} P &= 180.084 \text{ years} \\ T &= 1842.72 \\ e &= 0.129 \\ i &= 61^\circ.78 \\ \Omega &= 148^\circ.76 \\ \lambda &= 321^\circ.22 \\ a &= 4''.681 \end{aligned}\right\} \text{Elements of the First Approximation.}$$

For the purpose of effecting a least square solution, twelve normal places were next formed from the observations of the preceding table, as follows:

Date.	θ.	θ + Precession.	ρ.	*n.*
1852.48	156°.89	157°.13	3″.95	9–8
1857.31	152.95	153.17	4.45	2
1867.95	143.54	143.70	4.42	7–6
1874.83	134.55	134.67	4.12	4
1879.03	124.89	124.99	3.56	31
1882.54	118.35	118.44	3.37	13
1887.71	108.53	108.59	2.93	31
1891.13	99.01	99.05	2.46	18
1894.06	90.60	90.62	2.25	2
1898.54	75.81	75.82	2.50	5
1899.80	68.35	68.35	2.30	3
1900.92	63.41	63.41	2.40	2

From these there resulted twelve equations between the six unknown quantities, the residuals in angle only being employed. These equations were weighted and solved for the corrections to the elements, the results being as follows:

$$\left.\begin{aligned} P &= 180.039 \text{ years.} \\ T &= 1843.122 \\ e &= 0.133 \\ i &= 62^\circ.96 \\ \Omega &= 150^\circ.01 \\ \lambda &= 320^\circ.24 \\ a &= 4''.681 \end{aligned}\right\} \text{Elements of the Second Approximation.}$$

The residuals from these elements were not wholly satisfactory, especially between the years 1853 and 1879, when they steadily maintained the positive sign. For the purposes of a further improvement, therefore, the original observations were next grouped into the following thirty-three normal places:

Date.	ρ.	θ.	*n.*	*Observer.*
	″	°		
1850.94	3.96	156.6	2	O. S.
1851.28	3.58	156.7	3	O. S., Da.
1853.64	3.93	158.3	3	O. S.
1854.79	4.13	155.3	1	"
1856.80	4.51	152.9	1	"
1857.82	4.40	153.0	1	"
1864.84	4.45	147.6	2	Wi.,
1865.89	4.26	144.0	2	O. S.
1869.10	4.46	140.4	1	"
1871.99	2 ±	125±	1	Kn.
1872.56	4.62	140.6	1–2	O. S.
1873.99	4.27	133.9	1	"
1874.10	4.39	135.7	1	"
1875.14	3.80	138.1	1	"
1876.11	4.01	130.5	1	"
1877.86	3.80	127.5	11	D., C. O., B.
1878.14	4.36	125.5	1	O. S.
1879.09	3.50	125.3	6	B., Ha.
1880.52	3.22	121.6	10	B.
1881.84	3.53	119.0	6	B.
1882.12	3.25	118.2	2	Ha.
1883.40	3.09	117.5	4	B., Ha.
1884.16	3.74	118.2	1	H. S.
1886.30	3.04	111.9	11	L., Ha., T.
1888.51	2.95	106.7	11	Ha., B., T.
1889.12	2.79	103.6	4	Ha.
1890.84	2.27	99.6	7	B., Ho.
1891.38	2.57	98.1	9	B., Ha.
1893.21	2.18	93.8	1	Com.
1895.90	2.32	85.5	1–2	D., C.
1897.97	2.62	77.2	3	A.
1899.53	2.34	70.4	5	A., Doo.
1900.92	2.40	63.4	2	Doo.

These measures were corrected for precession, and to the resulting 33 equations there were assigned two series of weights, the first depending only on the number of nights, and the second being

arbitrarily assigned. Only the residuals in angle were employed, so that there resulted 33 equations between the six unknowns. The final values obtained from this solution led to the following elements:

$$\left.\begin{array}{lll} P &= 180.0288 & \pm 2.776 \text{ years.} \\ T &= 1843.185 & \pm 1.051 \\ e &= 0.13423 & \pm 0.0221 \\ i &= 63^\circ.25 & \pm 0^\circ.74 \\ \Omega &= 150^\circ.82 & \pm 0^\circ.71 \\ \lambda &= 319^\circ.54 & \pm 0^\circ.57 \\ a &= 4''.791 & \end{array}\right\} \text{The Final Elements.}$$

The value of a was obtained from a series of equations of the form

$$a = \cos(\Omega - \theta c) \sec(v + \lambda) \frac{fo}{(\iota - e \cos \varepsilon)}$$

The weighted mean was taken for the value of a.

The following table shows the agreement of the observed positions with the positions computed from the final elements. These residuals are perhaps as small as can be expected with a star of this character:

Date.	θc.	Prec.	θc—pr.	θo.	θo—θc.	ρc.	ρo.	ρo—ρc.	n.	Observer.
	°	°	°	°	°	″	″ ″			
1783.13	327.33	+ 0.58	326.75	326.7	± 0.00	5.14	4 to 8		1	H.
1825.12	234.22	0.37	233.85	287 ±		1.95			1	S.[1]
1835.86	189.97	0.32	189.65			2.61			1	S.[2]
1850.94	160.15	0.25	159.90	156.60	− 3.30	3.98	3.96	″	2	O. S.
1851.06	160.00	0.25	159.75	159.96	+ 0.21	3.99	3 ±	− 0.02	1	Dawes
1851.49	159.45	0.24	159.21	155.10	− 4.11	4.01	3.87	− 0.14	2	O. S.
1853.64	156.83	0.23	156.60	158.30	+ 1.70	4.09	3.93	− 0.16	3	O. S.
1854.79	155.47	0.23	155.24	155.30	+ 0.06	4.15	4.13	− 0.02	1	O. S.
1856.80	153.10	0.21	152.89	152.90	+ 0.01	4.22	4.51	+ 0.29	1	O. S.
1857.82	151.91	0.21	151.70	153.00	+ 1.30	4.22	4.40	+ 0.18	1	O. S.
1864.84	144.06	0.18	143.88	147.60	+ 3.72	4.26	4.45	+ 0.19	2	Winnecke
1865.89	142.85	0.17	142.68	143.95	+ 1.27	4.23	4.26	+ 0.03	2	O. S.
1869.10	139.06	+ 0.15	138.91	140.40	+ 1.49	4.12	4.46	+ 0.32	1	O. S.
1871.99	135.49	0.14	135.35	125 ±	−10.35	3.99	2 ±	− 1.99	1	Knott [3]
1872.56	134.80	0.14	134.66	140.65	+ 5.99	3.97	4.62	+ .65	1–2	O. S.
1873.99	132.89	0.13	132.76	133.90	+ 1.14	3.89	4.27	+ .38	1	O. S.
1874.10	132.74	0.13	132.61	135.70	+ 3.09	3.88	4.39	+ .51	1	O. S.
1875.14	131.31	0.12	131.19	138.10	+ 6.91	3.82	3.80	− .02	1	O. S.
1876.11	129.89	0.12	129.77	130.50	+ 0.73	3.75	4.01	+ .26	1	O. S.
1877.12	128.41	0.11	128.30	120 ±	− 8.30	3.69	2 ±	− 1.69	1	Flammarion
1877.84	127.36	0.11	127.25	127.5	+ 0.2	3.64	3.36	− .28	3	C. O.
1877.84	127.36	0.11	127.25	128.24	+ 1.01	3.64	3.92	+ .28	6	B.
1877.95	127.19	0.11	127.08	126.45	− 0.63	3.63	3.94	+ .31	4	D.
1878.14	126.89	0.11	126.78	125.50	− 1.28	3.62	4.36	+ .74	1	O. S.
1879.05	125.44	0.10	125.34	125.38	+ 0.04	3.56	3.49	− .07	4	B.
1879.18	125.22	0.10	125.12	125.00	− 0.12	3.55	3.52	− .03	2	Hall
1879.75	124.25	0.10	124.15	120.00	− 4.15	3.51	3.29	− .22	1	C. O.
1880.09	123.67	0.10	123.57	121.30	− 2.27	3.48	3.28	− .20	5	B.
1880.95	122.19	0.09	122.10	122.06	− 0.04	3.42	3.16	− .26	5	B.
1881.84	120.67	0.09	120.58	119.00	− 1.58	3.35	3.53	+ .18	6	B.
1882.12	120.15	0.09	120.06	118.15	− 1.91	3.33	3.25	− .08	2	Hall
1883.00	118.48	0.08	118.40	119.20	+ 0.80	3.27	3.07	− .20	2	B.
1883.81	116.89	0.08	116.81	115.80	− 1.01	3.21	3.10	− .11	2	Hall
1884.16	116.21	0.08	116.13	118.20	+ 2.07	3.18	3.74	+ .56	1	H. Struve
1886.00	112.13	0.07	112.06	112.15	+ 0.09	3.04	3.22	+ 0.18	2	Leavenw'th
1886.09	111.94	0.07	111.87	112.23	+ 0.36	3.03	3.00	− 0.03	6	Hall
1886.92	110.10	0.07	110.03	111.03	+ 1.00	2.97	3.01	+ 0.04	3	Tarrant
1887.14	109.61	0.06	109.55	109.18	− 0.37	2.95	2.56	− 0.39	4–1	Schiaparelli
1888.08	107.43	0.06	107.37	109.48	+ 2.11	2.88	2.26	− 0.62	2	Schiaparelli
1888.12	107.32	0.06	107.26	107.68	+ 0.42	2.88	3.04	+ 0.16	5	Hall
1888.84	105.52	0.06	105.46	106.83	+ 1.37	2.82	2.94	+ 0.12	3	B.
1888.87	105.45	0.06	105.39	105.05	− 0.34	2.82	2.81	− 0.01	3	Tarrant
1889.03	105.05	0.05	105.00	107.59	+ 2.59	2.81	2.87	+ 0.06	2–1	Schiaparelli
1889.12	104.82	0.05	104.77	103.55	− 1.22	2.81	2.79	− 0.02	4	Hall
1890.73	100.39	0.05	100.34	99.95	− 1.39	2.70	2.68	− 0.02	4	B.
1890.98	99.70	0.05	99.65	99.00	− 0.65	2.68	1.72	− 0.96	3	Hough
1891.01	99.62	0.04	99.58	101.49	+ 1.91	2.68	2.62	− 0.06	2	Schiaparelli
1891.06	99.49	0.04	99.45	98.56	− 0.89	2.67	2.65	− 0.02	5	Hall
1891.78	97.40	0.04	99.36	97.38	− 1.98	2.63	2.48	− 0.05	4	B.
1893.21	93.10	0.04	93.06	93.8	+ 0.70	2.55	2.18	− 0.37	1	Comstock
1895.89	84.17	0.02	84.15	83.65	− 0.50	2.43			1	Doberck [4]
1895.91	84.14	0.02	84.12	87.4	+ 3.30	2.43	2.32	− 0.11	1	Collins
1897.97	76.73	0.01	76.72	77.22	+ 0.50	2.36	2.62	+ 0.26	3	Aitkin [5]
1899.11	72.49	0.00	72.49	73.6	+ 1.10	2.33	2.39	+ 0.06	2	Aitkin
1899.80	69.50	0.00	69.50	68.35	− 1.15	2.32	2.30	− 0.02	3	Doolittle
1900.92	65.61	0.00	65.61	63.41	− 2.20	2.31	2.40	+ 0.09	2	Doolittle
1903.14	57.19	° .02	57.21	55.22	− 1.99	2.34	2.46	+ 0.12	4	Doolittle

[1] "A very vague estimate" (O. S.).
[2] "No trace of duplicity."
[3] This is a rough estimate merely; Knott used a 7⅓-inch.
[4] "Nearly invisible."
[5] 12-inch.

There have been two determinations made of the parallax of this star; the first determination was by the heliometer by Gill in 1882, and the second was by micrometric measures by Hall in 1884. The results were:

Gill,	$0''.16$	19.6 light years.
Hall,	$0''.22$	14.6 light years.

If we assume the mean of these, or $0''.19$, as the most probable value, the dimensions of the orbit and the combined mass of the two components can readily be determined. We find that the sum of the masses of the two components is nine-tenths the mass of our sun, and that the semi-major axis of the true orbit is 23.5 times the distance from the earth to the sun. The orbit is thus larger than the orbit of Uranus, but inferior to that of Neptune.

UNIVERSITY OF PENNSYLVANIA, PHILADELPHIA.

SOME ABORIGINAL LANGUAGES OF QUEENSLAND AND VICTORIA.

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MEMB. ASSOC. ETRAN. SOC. D'ANTHROP. DE PARIS.

(Read October 3, 1902.)

Last year I contributed to this Society a short description of the Gundungurra, one of the native tongues of New South Wales. In the following pages it is proposed to furnish the outlines of the grammatical structure of some aboriginal languages spoken by the native tribes of Queensland and Victoria.

The method of spelling adopted is that recommended by the Royal Geographical Society of London, with the following qualifications:

As far as possible vowels are unmarked, but in some instances the long sound of a, e, and u are indicated thus, ā, ē, ū. In a few cases, to avoid ambiguity of pronunciation, the short sound of u is thus represented, ŭ.

G is hard in all cases. R has a rough, trilled sound, as in "hurrah!" W always commences a syllable or word. Y at the beginning of a word or syllable has its ordinary consonant value.

The sound of the Spanish ñ often occurs; at the beginning of a